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AIRBORNE RADAR SEARCH FOR DIESEL SUBMARINES (ARSDS)

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Outline

- Acknowledgement
- Context
- Analytical approach
- Search theory review
- ARSDS detection rate model
- Example & Insights

Acknowledgements

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  - *Radar Search and Detection with the CASA 212 S43 Aircraft*

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    Thesis Advisor: Steven E. Pilnick

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- Related Work
  - Tactics Development & Evaluation (TAC D&E) Program

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  - Other thesis work

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Context

- **Airborne Radar Search For Diesel Submarines**
  - Diesel submarine detection is challenging
    - Active sonar limited by short ranges
    - Passive sonar limited by quietness of submerged diesel submarines while on battery propulsion
  - Airborne search is an historical, preferred tactic
    - Catch a submarine when it is on the surface or with masts or periscopes exposed, briefly or intermittently, for battery charging, communications, or surface surveillance
  - Issue
    - In the past, tactics have been based on operational judgment -- guesses about effective search area, etc.

Analytical Approach

- A Detection Rate Model is developed for ARSDS
  - MOE: probability of radar detection of a submarine that is only detectable during intermittent periods of periscope exposure
- Use of the model
  - evaluate search tactics effectiveness as a function of
    - search area, searcher altitude, number of search aircraft, etc.
  - use the model as an aid to understanding effects of changes in
    - submarine operating profile, radar cross section, etc.

Search Theory Review

Detection Rate Models

- Used for modeling probability of detection for *continuous-looking search*
- The detection process is a Poisson Process
  - independent increments, etc.
  - constant detection rate $\lambda \Rightarrow$
    - Poisson # detections in time *t*
    - exponential times between detections; etc.

$$P\{1 \text{ or more det in time } t\} = 1 - e^{-\lambda t}$$

  - variable detection rate $\gamma(t) \Rightarrow$
    - non-homogeneous Poisson Process

$$P\{1 \text{ or more det in time } t\} = 1 - e^{-\int_0^t \gamma(s)\,ds}$$

Search Theory Review
Detection Rate Models

- Examples
  - Inverse-cube Law of Sighting (visual search)
  - Poisson scan model (sonar search)
  - Blip-scan model (radar search)
  - Random search model
    - constant detection rate = vw / A
    - where v = search speed, w = sweep width, A = search area

$$P\{1 \text{ or more det in time } t\} = 1 - e^{-\frac{vw}{A}t}$$

- The key to detection rate models is coming up with a detection rate

ARSDS Detection Rate

- Idea
  - The rate at which detections can be made is governed by the rate at which occasional periscope exposures occur
  - When an exposure occurs, it can result in detection if
    - the searching aircraft radar happens to be covering the patch of ocean where the submarine periscope happens to be, and
    - the submarine does not get a chance to evade due to radar counter-detection

ARSDS Detection Rate

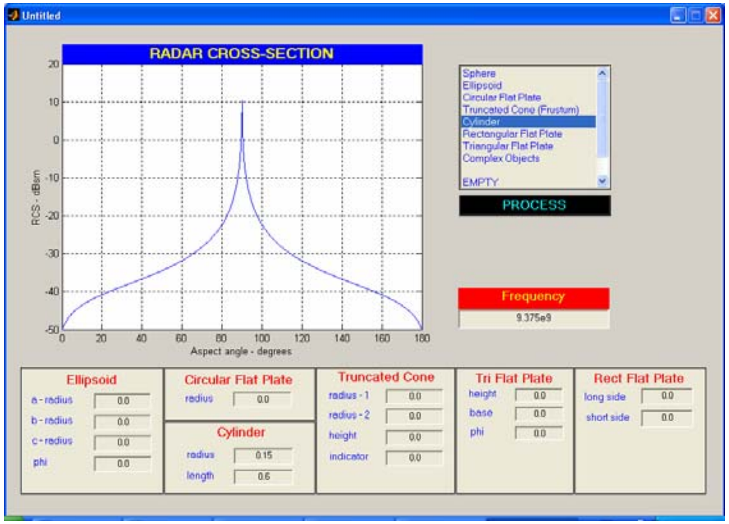
$$\begin{array}{c}\text{ARSDS}\\ \text{Detection}\\ \text{Rate}\end{array} = \begin{bmatrix}\text{Rate of}\\ \text{occurrence}\\ \text{of submarine}\\ \text{periscope}\\ \text{exposures}\end{bmatrix} * P\begin{bmatrix}\text{Aircraft radar}\\ \text{detection patch}\\ \text{is covering spot}\\ \text{when periscope}\\ \text{exposure occurs}\end{bmatrix} * P\begin{bmatrix}\text{Submarine does}\\ \text{not avoid}\\ \text{detection}\\ \text{due to radar}\\ \text{counter-detection}\end{bmatrix}$$

Model Parameters

- Submarine
  - typical frequency and duration of required operations with periscopes or masts exposed
  - periscope & mast radar cross section (RCS)
- Radar
  - detection range vs. RCS vs. search altitude
  - counter-detection range
  - sea-state detection degradation
- Search Aircraft
  - search area & number of aircraft
  - search speed
  - search altitude

Periscope RCS

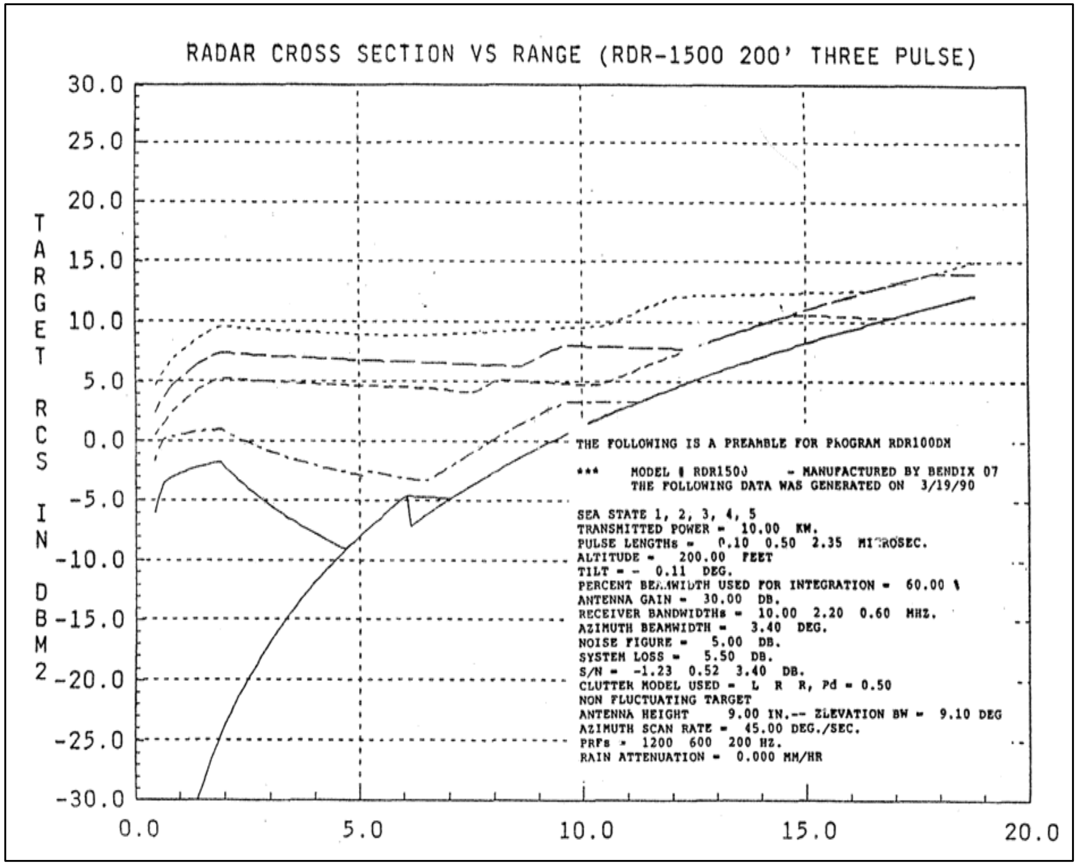
- Preferred: actual target RCS data
  - Use if available
- Alternative: computed RCS
  - normal radar reflection RCS computed with a Physics model
    - height & shape of exposed mast assumed
    - actual search radar frequency
  - degradation from perfect reflection due to sea-state
    - uses an assumed % reflection table



SEA STATE		
Sea State	**Correction factor**	**Condition**
0	100.00%	Flat Surface
1	90.00%	Smooth
2	75.00%	Slight
3	50.00%	Moderate
4	15.00%	Rough
5	2.50%	Very Rough
6	1.00%	High
7	0.15%	Very High
8	0.07%	Mountainous
9	0.01%	Very Mountainous

Search Radar Max Range

- Actual Radar Manufacturer Data
  - based on radar range equation
  - input
    - target RCS
    - aircraft altitude
    - radar mode, settings, etc.
  - look-up
    - radar maximum detection range

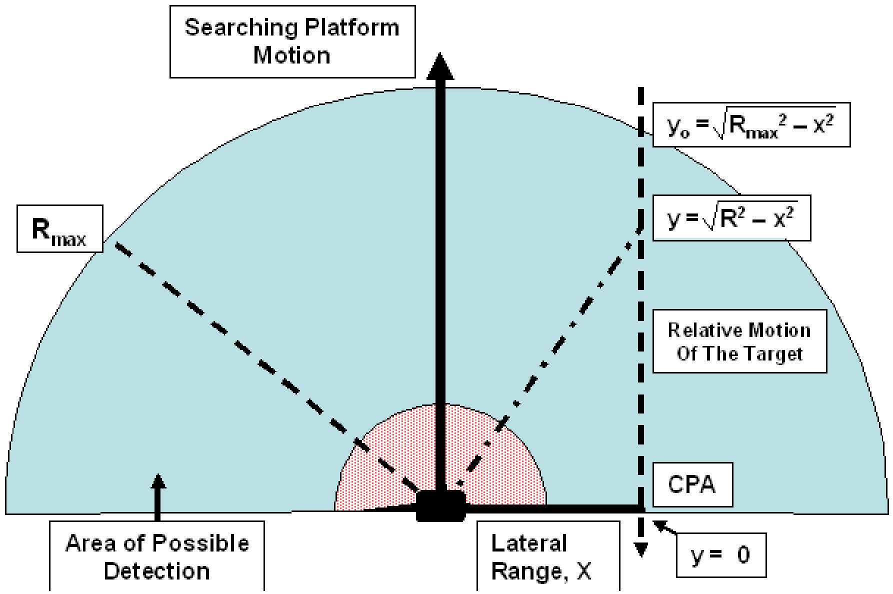


Effective Radar Sweep Width

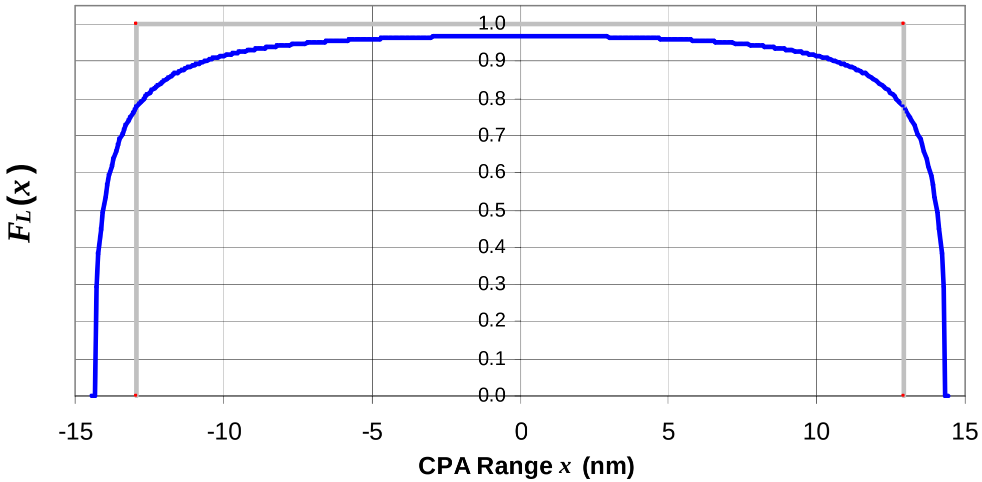
- Sweep Width, w
  - Based on search theory
  - Not assumed cookie-cutter
  - Derive sweep width by integrating the radar lateral range function over all possible CPA ranges

$$w = \int_{-R_{\max}}^{+R_{\max}} F_L(x)\, dx$$

- Lateral Range Function
  - Preferred: Empirical data from live operational radar detection testing
  - Alternative: Derive a lateral range function from a simple geometric model and assumed scaling



Lateral Range Function
Small Target



Swept Radar Patches

- Sweep rate (definition)

sweep rate = sweep width (w) * aircraft search speed (v)

- Patch length

For convenience, we say that the radar lays down a pattern of non-overlapping patches, each considered a radar glimpse

$$\text{Patch Length} = R_{\text{max}} - R_{\text{min}}$$

- Radar Glimpse Interval (calculated)

defined as the time it takes the aircraft to fly over one radar coverage patch

$$\text{Radar Glimpse Interval (hrs)} = \frac{\text{Radar Patch Length (nm)}}{\text{Aircraft Search Speed (kts)}}$$

Conditional Glimpse p_d

- Radar patch area
  - increment of area swept by the search aircraft in one glimpse interval

$$\text{Radar Patch Area} = \begin{pmatrix} \text{Radar} \\ \text{Glimpse} \\ \text{Interval} \end{pmatrix} * \begin{pmatrix} \text{Effective} \\ \text{Sweep} \\ \text{Rate} \end{pmatrix}$$

- Conditional Glimpse p_d
  - the likelihood that the relatively small aircraft radar patch happens to be covering the point in the much larger search area, A, when a detection opportunity (i.e., periscope exposure) occurs.
  - It is assumed that the uncertain submarine position, when exposed, is equally likely to be anywhere in the search area, A.

$$\text{Conditional Glimpse } p_d = \frac{\text{Radar Patch Area}}{\text{Search Area}}$$

Periscope Exposure Rate (1)

- Operational Period (user input)
  - any convenient fixed time period used to summarize the submarine operating profile, such as a 24-hour day
  - includes time spent completely submerged and time spent with periscopes or masts exposed for any purpose
  - user input
- Periscope Exposure Hours (user input)
  - expected amount of time during each Operational Period that the submarine has periscopes or masts exposed for any purpose such as recharging batteries, communicating, or conducting surveillance

Periscope Exposure Rate (2)

- Glimpse Count (calculated)
  - counts the number of glimpse intervals that comprise Periscope Exposure Hours during each Operational Period

$$\text{Glimpse Count} = \frac{\text{Periscope Exposure Hours (hrs)}}{\text{Radar Glimpse Interval (hrs)}}$$

- Periscope exposure rate (calculated)

$$\text{Periscope Exposure Rate (hrs}^{-1}\text{)} = \frac{\text{Glimpse Count}}{\text{Operational Period (hrs)}}$$

Note: This version of the model computes a constant periscope exposure rate (or detection opportunity rate). The model could be easily adapted to allow for an opportunity rate that varies by time of day, for example.

ARSDS Detection Rate

- So far, neglecting radar counter-detection

$$\begin{array}{c}\text{ARSDS}\\ \text{Detection}\\ \text{Rate}\end{array} = \left[\begin{array}{c}\text{periscope}\\ \text{exposure}\\ \text{rate}\end{array}\right] * p_{d|\text{exposure}}$$

$$= \frac{\left(\dfrac{\text{Periscope Exposure Hours}}{\text{Radar Glimpse Interval}}\right)}{\text{Operational Period}} * \frac{\left(\begin{array}{c}\text{Radar}\\ \text{Glimpse}\\ \text{Interval}\end{array}\right) * \left(\begin{array}{c}\text{Effective}\\ \text{Sweep}\\ \text{Rate}\end{array}\right)}{\text{Search Area}}$$

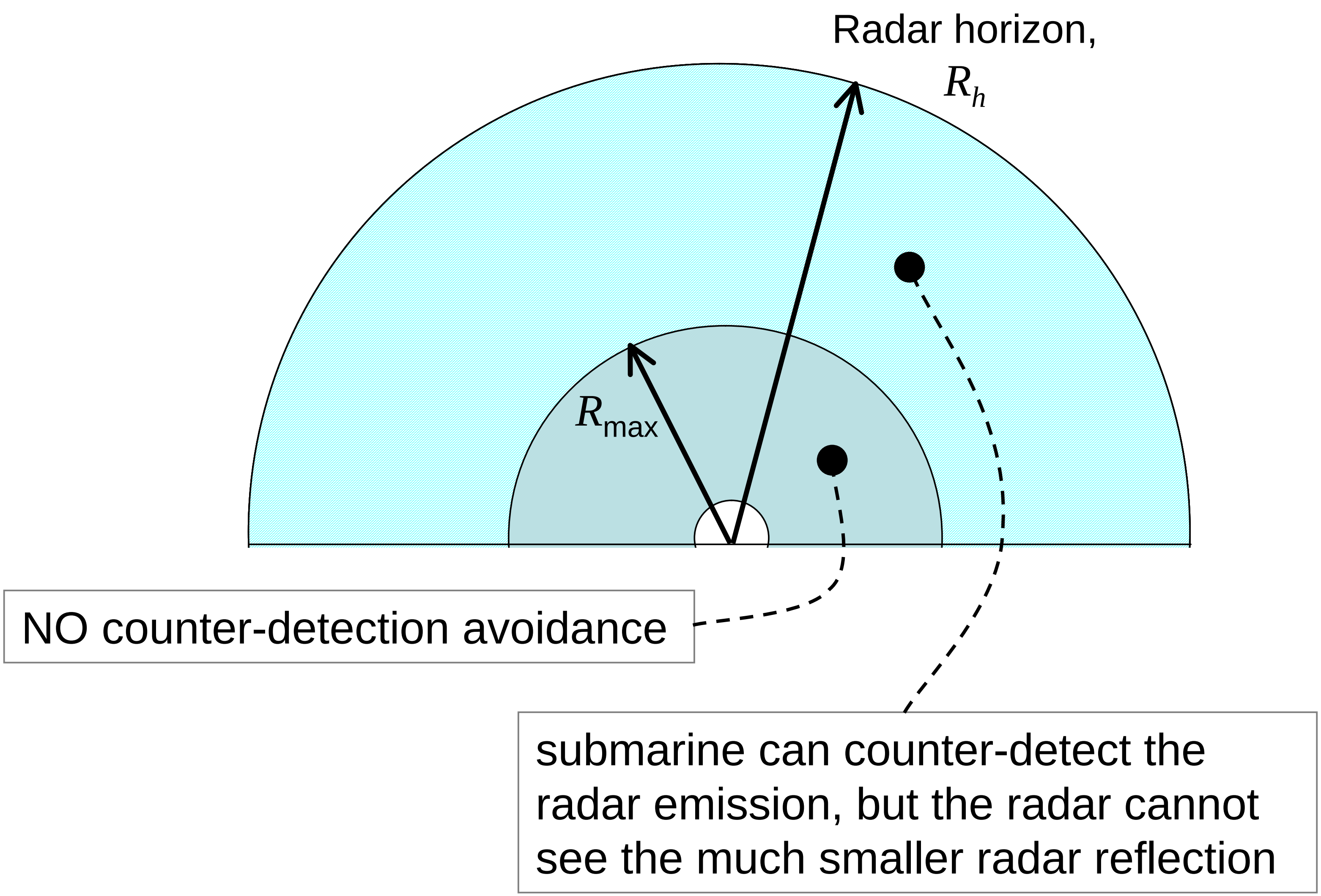
$$= \left(\frac{\text{Periscope Exposure Hours}}{\text{Operational Period}}\right) * \frac{v\,w}{A}$$

$$= p_{\text{exposure}} * \frac{v\,w}{A}$$

ARSDS detection rate is a fraction of the random search model detection rate !

Radar Counter-detection by the Submarine

- The model considers the possibility that the search radar can be counter-detected by the target submarine



Radar Counter-detection by the Submarine

- The conditional probability that a submarine within the search aircraft radar horizon does not get the chance to avoid detection due to radar counter detection is modeled as the ratio of the detection area to the horizon area, or

$$P\begin{bmatrix}\text{Submarine does not} \\ \text{avoid detection due to} \\ \text{radar counter-detection}\end{bmatrix} = \frac{{R_{\text{max}}}^2}{{R_h}^2}$$

ARSDS Detection Rate Model

- ARSDS Detection Rate

$$\text{ARSDS Detection Rate} = \frac{v\,w}{A} * p_{\text{exposure}} * \frac{R_{\max}{}^2}{R_h{}^2}$$

- ARSDS Cumulative Detection Probability

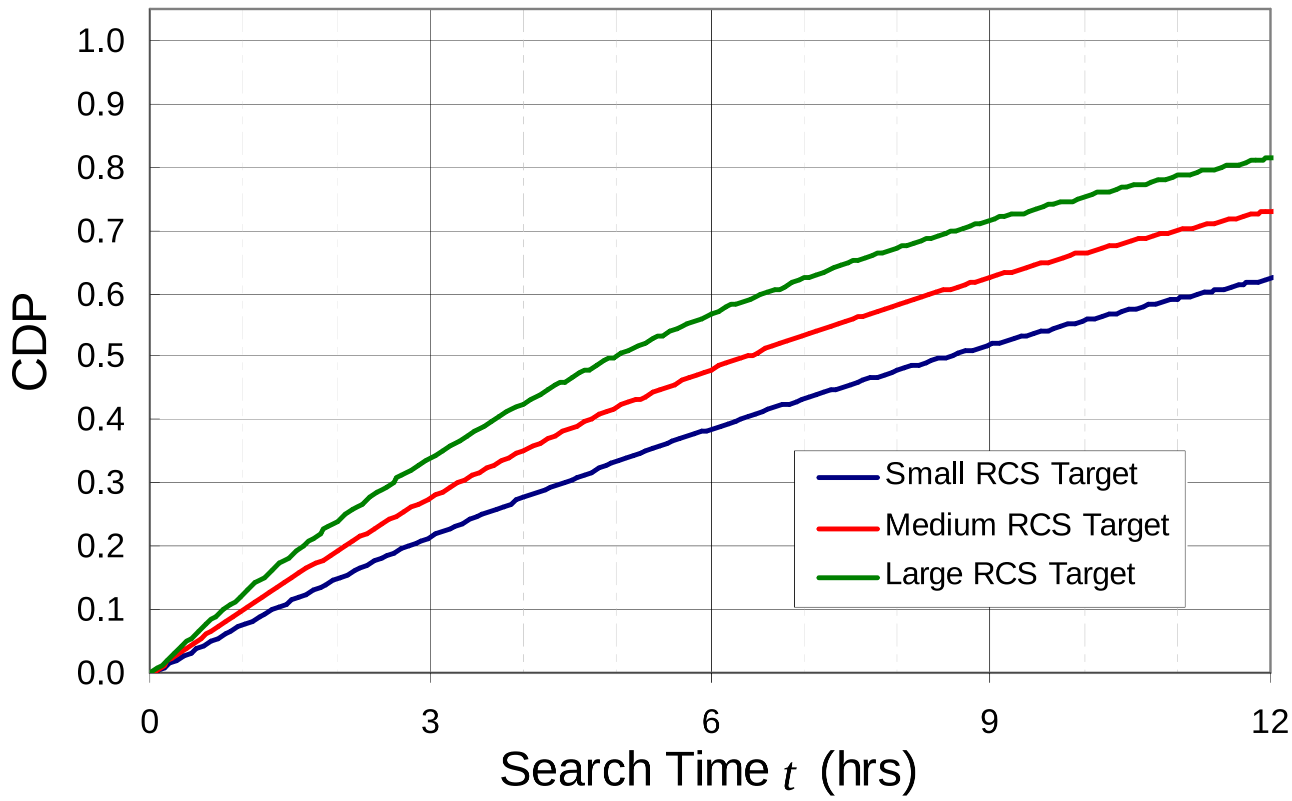
$$P\{1 \text{ or more det in time } t\} = 1 - e^{-\frac{v\,w}{A} p_{\text{exposure}} \frac{R_{\max}{}^2}{R_h{}^2} t}$$

Note: The model could be easily adapted to allow for a situation where the detection rate varies by time of day

Example

Exposed periscope heights: .5, .6, .7 m
Total exposure per day: 6 hrs
Sea State 1

Search area: 60 x 60 nm
Aircraft altitude: 500 ft
Aircraft speed: 180 kts



Effects of Aircraft Altitude

Operational Insight from the Model:

- Low aircraft altitude improves ARSDS detection rate two ways
  - low altitude increases the maximum detection range against small RCS targets, and
  - low altitude shortens the distance to the radar horizon, and thus reduces the chance that a submarine can take advantage of a counter-detection
- Unfortunately, low altitude also does one other thing
  - low altitude decreases aircraft fuel efficiency thus reducing flight endurance
- Therefore, there is a tradeoff of flight endurance for detection probability

Sea-State Degradation of RCS

Operational Insight from the Model:

- For fixed periscope exposure height, increasing sea-state has the effect of decreasing target RCS
  - Decreased RCS shortens maximum detection range, causing two penalties .
    - First, the sweep width is reduced, which by itself diminishes the detection rate
      - sweep width is proportional to max range
    - Secondly, the shortened radius of the maximum detection area increases the chance that the submarine can avoid detection entirely due to counter-detection evasion, which causes detection rate to diminish further
      - probability sub avoids due to counter-detection is proportional to R_{max}^2
- Combined, detection rate is approximately proportional to R_{max}^3
  - Example: If diminished RCS decreases maximum detection range by 50% (i.e. to 1/2 of the previous maximum detection range) then the detection rate is reduced to $(1/2)^3$ or 1/8th of the previous detection rate
- The operational implication of this is that as sea-state increases, the aircraft search plan may need to compensate for the reduced RCS with **much** smaller search areas and **lower** search altitudes

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- Acknowledgement
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- ARSDS detection rate model
- Example & Insights

Backups

Abstract

Aircraft search to catch diesel submarines on the sea surface or with masts exposed above the sea surface has been an anti-submarine warfare tactic for more than half a century. However, rather than analysis, operational judgment has been used to guess at good search tactics such as how large an area can one aircraft cover effectively. In this research, a detection rate model is developed to analyze the effectiveness of an airborne radar search for a diesel submarine assumed to be intermittently operating with periscopes or masts exposed above the sea surface. The analysis obtains cumulative probability of detection vs. time based on the radar manufacturer’s performance data, user inputs for aircraft search area size, search speed, and search altitude, and submarine periscope or mast exposure profile. The model can use given periscope radar cross section data, or roughly calculate radar cross section given assumptions about exposed periscope height above the sea-surface and sea-state conditions. Submarine evasion due to radar counter-detection is also modeled.

Additional Notes

- Periscope Exposure Rate
  - In actual practice, a submarine might use different periscopes or masts for each function. For the sake of simplicity, the current version of this model assumes one common periscope/mast for all functions and aggregates the total time exposed per period. The model could be expanded to consider different periscopes or masts (with different radar cross sections) exposed for differing amounts of time. If different masts were modeled, then it would be appropriate to distinguish exposure times for each unique periscope-mast configuration.

Additional Notes

- Submarine Speed
  - The model does not explicitly use submarine speed as an input
  - Submarine speed does implicitly determine the rate at which the submarine needs to recharge batteries, which is used in the model

Sweep Width and Lateral Range Function

Sweep width for the radar when flown at a particular altitude searching for a target of a particular radar cross section is needed for computing the detection rate. Two options exist for determining sweep width.

a. Option One

Option one would assume the radar footprint acts like a cookie-cutter and thus the overall width of the footprint would be the sweep width. The following discussion describes the reasoning behind this method and concludes that it is not used due to some shortcomings.

Since the radar footprint was determined based upon the radar ability to see targets within that footprint (and conversely its inability to see targets outside the footprint), the radar footprint could possibly be interpreted as a cookie-cutter detection pattern (i.e., detecting every target that falls within the footprint with probability 1).

Such a cookie-cutter sweep width might be overly optimistic in practice because of the irregular shape of the radar footprint. In fact, as the radar footprint sweeps over area, points close to the extreme left and right corners of the pattern are within the footprint for much less time than points that are passed closer to the middle of the pattern.

Accordingly, it is deemed unrealistic to treat the full width of the radar footprint as a cookie-cutter sweep width, and therefore this method is not used.

b. Option Two

Option Two is to calculate sweep width as the integral of the lateral range function over all possible closest points of approach between the aircraft and the submarine (i.e., find the area under the radar lateral range curve). This is the preferred method that is used.

If actual lateral range curves for the radar were available from the manufacturer, or from operational testing, they could be used directly. However, lacking such data, a lateral range function can be approximated based on the geometry of the radar footprint and the proportional amount of time that an exposed target will fall within the footprint as a function of the closest point of approach between the exposed target and the aircraft.

Lateral Range Function

Lateral range is the closest point of approach (CPA) between the searcher and the target assuming an infinitely long straight line relative motion path.

The lateral range function, $F_L(x)$, is a cumulative detection probability as a function of the lateral range x.

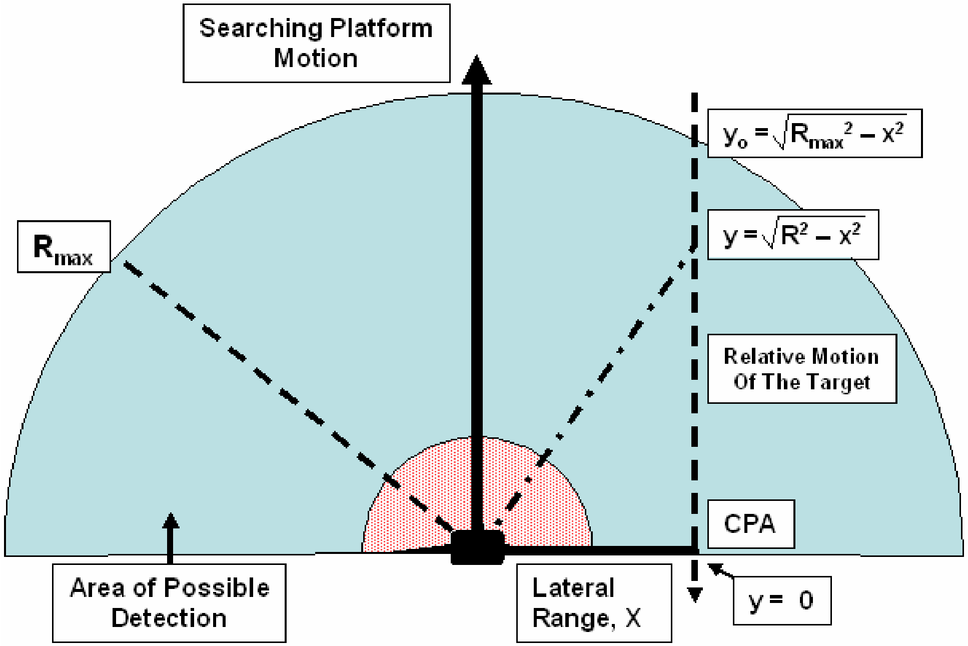
These definitions implicitly assume that a target exists that can be detected. In this context, the target would be an exposed submarine periscope. Accordingly, the cumulative probability of detection used in the lateral range function might more correctly be called a conditional cumulative probability of detection given that the submarine periscope is exposed. This is very significantly different from the cumulative detection probability that is ultimately computed based on intermittent submarine periscope exposure and counter-detection evasion.

Ref: Wagner, et.al, *Naval Operations Analysis*, 3rd Ed., Naval Institute Press, 1999

Lateral Range Function Derivation

- Detection depends on the maximum detection range (Rmax), the amount of time an exposed target would be inside the radar footprint, and whatever the detection rate is for an exposed target.
- When CPA range $x \le$ Rmax, the target could be detected and when CPA range $x >$ Rmax the target is not detectable.
- The target enters in the area of possible detection at point (x, y_0). The location of the target at time t is $(x, y(t)) = (x, y_0 - vt)$, where v is the relative speed.
- Submarine speed is very slow compared to aircraft search speed and thus relative speed is approximately just the aircraft speed.
- The target reaches CPA at time $t = y_0 / v$ and moves out of the area of detection.

Ref: Wagner, et.al, *Naval Operations Analysis*, 3rd Ed., Naval Institute Press, 1999



Lateral Range Function Notes

Wagner derives a lateral range function for a situation comparable to the situation here. If it is assumed that the radar footprint passes over an area containing an exposed submarine periscope, and that during this encounter a constant detection rate applies, then the lateral range function takes the following form, where K is a constant.

$$F_L(x) = 1 - e^{\left(-K\sqrt{R_{max}^2 - x^2}\,/\,v\right)} \qquad \text{for } x \le R_{max}$$

The maximum value of this lateral range function, when CPA range x = 0, is

$$P_{\max} = 1 - e^{\left(-K\,R_{\max}\,/\,v\right)}$$

From this an expression is obtained for the constant K.

$$K = -\left(\frac{v}{R_{\max}}\right)\ln\left(1 - P_{\max}\right)$$

We treat the parameter $P_{\max}$ as a scaling parameter to generate an approximate lateral range function that is deemed to be realistic for the given radar and given target radar cross section.

Reminder: This derived lateral range function is a model placeholder for the radars empirical lateral range function based on data, if it were available.

Effective Sweep Width

Sweep width, w is defined as the area under the lateral range curve

$$w = \int_{-R_{\max}}^{+R_{\max}} F_L(x)\, dx$$

It is common to also think about a cookie-cutter sensor that has the same sweep width, that in some circumstances may provide equivalent performance

The lateral range function of the “equivalent” cookie-cutter sensor is rectangular, with height 1.0 and width w

This interpretation corresponds to the common understanding about sweep width representing a definite swath of detection swept out by the sensor

Sweep Width for a Small Target